Position Estimation from Range Measurements Using Adaptive Networks

Guilherme S. Vicinansa, Yannick P. Bergamo, Cassio G. Lopes
Electrical Engineering, University of Sao Paulo
Escola Politecnica, Sao Paulo, Brazil, 05508-970

Abstract—In this paper we introduce a new multilateration-based method for source localization. Sensors with known positions collect noisy signals whose strength depends on the relative position between the sensors and the source. A nonlinear system of equations is obtained which is then recast into a linearized least squares (LS) problem, which resembles a multilateration scenario. From the LS problem a low-complexity Adaptive Network (AN) solution is proposed via a distributed Diffusion Least Mean Square (LMS) implementation. Two well known signal strength models are reformulated to fit the AN framework. The resulting distributed solution inherits the robustness, low complexity and intrinsic tracking ability of ANs. Simulations show the effectiveness of the new method in noisy scenarios when localizing stationary sources and when tracking moving targets, even for a small number of sensors.


I. INTRODUCTION

Adaptive networks (ANs) consist in a network of cooperative adaptive filters (AFs) distributed across a field of interest that collectively attempt to estimate a global unknown vector of parameters [1]. Each AF captures local signals and cooperates with its direct neighbors in order to generate a local estimate of the unknown vector [1]–[2].

ANs have been recently used in many applications, including self-organization [3], cognitive radio networks [4] and scalar field estimation [5]. The challenge is to pose the problem in the AN framework, which assumes that two signals are available at each sensor node and that they are related via a noisy linear model.

The problem of source localization based on scalar measurements is important in a myriad of applications, from speaker localization to structural monitoring [6]–[8]. Different approaches are available with different complexity and performance, some of which are distributed [6], [9]. In this work we develop an AN solution to the problem of source localization by formulating a multilateration problem [10]. Two well known signal models for the field of interest are recast in the AN formulation [1], which presents low complexity, robustness and natural tracking ability inherited from its cooperative AFs. Several simulation scenarios illustrate, under low and high SNR, the ability of the proposed AN to estimate stationary sources. Tracking a moving target following two different dynamic models is also explored, considering inaccuracies at the nodes positions.

II. THE MULTILATERATION PROBLEM

The multilateration problem is at the core of a number of applications. From GPS location to mechanical structures monitoring, it is widespread in engineering and related fields. It consists of finding the position $\theta \in \mathbb{R}^D$ of an object or an event, where $D$ is the dimension of the space where the localization takes place, having available only range measurements from $N$ points$^1$ with known positions relatively to the coordinate system where the source is to be located [10]. The range reading $\rho_k$ at node $k$ follows a function $g$ of the nodes positions $p_k$ and the source position $\theta$ that is application dependent

$$\rho_k = g(p_k, \theta) \quad k = 1, \ldots, N$$

A simple yet relevant scenario to motivate the use of ANs is the one in which the range reading is proportional to the squared distance to the source, i.e., $g(p_k, \theta) = \|p_k - \theta\|^2$, so that given the positions $p_k$ where the $N$ ranges readings $\rho_k$ are obtained, we want to find the position $\theta$ of the object that satisfies

$$\|p_k - \theta\|^2 = \rho_k^2, \quad k = 1, \ldots, N$$

This is a nonlinear system of equations with possibly more equations than unknowns, that in general will not have a solution since the measurements $\rho_k$ are noisy. Inspired by [11], we subtract one equation, e.g., the $N^{th}$, from all the others, leading to a linear system of $N - 1$ equations that may be recast in the AN formulation

$$-2(p_k - p_N)^T \theta = \rho_k^2 - \rho_N^2 - \|p_k\|^2 + \|p_N\|^2, \quad k = 1, \ldots, N - 1$$

III. ADAPTIVE NETWORK SOLUTION

We assume a network of $N$ nodes, geographically distributed with positions$^2$ $p_k$ for $k = 1, \ldots, N$, whose realizations $p_{k,i}$ are assumed to be known by each node $k$ at each time step $i$. Each node has also access to local signals $u_{k,i} \in \mathbb{R}^{1 \times M}$ and $d_k(i) \in \mathbb{R}$ which are realizations of the random quantities $\{u_k, d_k\}$. We further assume that each node $k$ is able to transmit to and receive data from its neighbors.

$^1$It is necessary to have at least $D + 1$ points in order to this problem to have a unique solution.

$^2$As in [12], boldface symbols represent random quantities, so that $p_k$ can account for drift or motion control, and it is assumed independent from other random quantities.
described by the set $\mathcal{N}_{k,i} = \{ l : \| p_{k,i} - p_{l,i} \| < s \}$, for some $s > 0$; we also define $\bar{\mathcal{N}}_{k,i} = \mathcal{N}_{k,i} \setminus \{ k \}$.

The task of the network is then to find an optimal weight $w_0$ that minimizes the following cost function [1]

$$ J(w) = \sum_{k=1}^{N} E[|d_k - u_kw|^2] $$

(4)

Several algorithms can be used to solve this problem adaptively and in a distributed fashion [1], [2], [13]–[15]. Here we consider the standard Diffusion LMS algorithm [1]

$$ w_{k,i-1} = \sum_{l \in \mathcal{N}_{k,i}} c_{kl} \psi_{l,i-1} $$

(5a)

$$ \psi_{k,i} = w_{k,i-1} + \mu_k w_{k,i} (d_k(i) - u_k w_{k,i-1}) $$

(5b)

The vector $u_{k,i}$ and the signal $d_k(i)$ are application dependent. They must be expressed in terms of quantities that reflect the localization problem, in other words, in terms of $p_{k,i}$ and of a signal $r_k(i)$ which is an estimate of the squared distance to the object $z_k$

$$ r_k = \| p_k - \theta \|^2 + z_k $$

(6)

where $z_k$ represents measurement noise.

The first step is to extend (3) by subtracting from node $k$’s equation a convex combination of its neighborhood’s equations, instead of only one equation. This allows for aggregating spatial information and considerably improves network performance [1], [2]. As such, in order to simplify the presentation, we introduce the notation $\mathcal{D}_k$, such that given a quantity, vector or scalar, $q = (q_1, \ldots, q_N)$, where $q_i \in \mathcal{N}_k$ is available to node $k$, and a function $f$ applied to it

$$ \mathcal{D}_k f(q) \triangleq f(q_k) - \sum_{l \in \mathcal{N}_k} a_{kl} f(q_l) $$

(7)

where $\{a_{kl}\}$ form a set of convex combiners, i.e., $a_{kl} \geq 0$ and $\sum_{l \in \mathcal{N}_k} a_{kl} = 1$ and function $f$ is applied element-wise over its argument. For example, we have $\mathcal{D}_k \|p\|^2 = \|p_k\|^2 - \sum_{l \in \mathcal{N}_k} a_{kl} \|p_l\|^2$. Throughout the text, whenever $\mathcal{D}_k$ appears more than once in the same expression it is implicitly assumed that the same set of combiners is used. Each node will then compute its desired signal $d_k$ from

$$ d_k = \mathcal{D}_k r - \mathcal{D}_k \|p\|^2 $$

(8)

Introducing data model (6) into (8) and rearranging terms we obtain

$$ d_k = -2(\mathcal{D}_k p)^T \theta + \mathcal{D}_k z $$

(9)

We can then define the regressor

$$ u_k = -2(\mathcal{D}_k p)^T $$

(10)

and thus rewrite (8) as

$$ d_k = u_k \theta + v_k $$

(11)

As mentioned earlier, $p_{k,i}$ and $r_k(i)$ are space-time realizations of the random processes $p_k$ and $r_k(i)$.

where $v_k = \mathcal{D}_k z$. Thus, we conclude that the multilateration problem can be recast into the general AN framework.

At each time iteration $i$, node $k$ generates its regressor via (10), measures the range measurement $r_k(i)$ and generates its desired signal from (8). The local adaptive estimate $w_{k,i}$ for $\theta$ is obtained via the well known adaptive equations (5).

Other schemes are available in the literature. In [3], a scheme is proposed where each node has available an estimate of the relative distance and bearing of the object. In [9] an ANLS based method is proposed for acoustic source localization. However, there are constraints in the sensors positioning and the noise average is assumed as known. Also, there is no evidence that the algorithm works for moving targets. In [6] an approach similar to the multilateration problem is studied but the solution requires a central node and an initial (reliable) position estimate. Finally, in [7] the Expected-Maximization method is proposed but processing is not in-network, and a reliable initial estimate is required for good performance. Our solution is fully distributed, adaptive and low-complexity and requires only a scalar measurement and the knowledge of the nodes position, which is a typical assumption. Due to the structural difference and (much) higher complexity, a direct comparison is not fair. In addition, the AN formulation may be initialized with any initial condition.

In the source localization problem the AN performance is better described by the mean-square deviation (MSD) in the local filter weights

$$ MSD_k(i) = E\|w_{k,i-1} - w^*\|^2 $$

(12)

where $MSD_k(i)$ captures the local performance (node level) and the global (network) performance $MSD(i)$ is obtained averaging (12) over $N$ nodes.

The simulation scenario is built such that nodes are randomly deployed following a uniform distribution in the beginning of each trial over a $10 \times 10$ grid, and then kept fixed. The object position is set to $\theta = [2.5, 4]^T$, each node’s estimate is initialized at zero, and the additive noise in (6) was generated using a Gaussian distribution with $\mathbb{E} z_k = 0$ and variance $\sigma^2 = 0.01$.

Figure 1 shows the AN evolution for small networks ($N = 3$ is the smallest number for the bidimensional case). Note how the technique is able to find the correct source position. Also, as expected, performance improves as spatial diversity is further explored (as $N$ increases). Figure 1 also shows one realization of the estimate $w_{k,i}$ for some node in the $N = 10$ scenario.

IV. NORM PROPORTIONAL SIGNAL MODEL

When the signal is proportional to the distance (i.e., not squared), which is the case of sensors that use time to measure distance [16], the technique developed in the previous section may be promptly accommodated. Consider the signal received at the node $k$ as

$$ r_k = \| p_k - \theta \| + z_k $$

(13)
and assume that the noise $z_k$ is a Wide Sense Stationary (WSS) process with zero mean and a known variance $\sigma^2_k$. Then squaring signal $r_k$ results

$$r_k^2 = ||p_k - \theta||^2 + 2||p_k - \theta||z_k + z_k^2 \tag{14}$$

Assuming that a noise variance estimate $\hat{\sigma}^2_k$ is provided and subtracting it from the squared signal (14) results in an auxiliary signal

$$s_k \triangleq r_k^2 - \hat{\sigma}^2_k \tag{15}$$

whose underlying signal model is equivalent in the mean to the original signal model (6), i.e.,

$$E[s_k] = E[||p_k - \theta||^2 + 2||p_k - \theta||E[z_k] + E[z_k^2] - \hat{\sigma}^2_k] \tag{16}$$

The noise is zero mean, thus the second term in the right hand side is zero. Furthermore, from the definition of variance $E[z_k^2] = \sigma^2_k$. Thus, if the variance estimate is accurate

$$E[s_k] = E[||p_k - \theta||^2] \tag{17}$$

Note that both models (6) and (13) imply that SNR varies over space, as nodes are deployed at different distances from the source. Note also that this problem is exactly the same as in the squared case, just with a different variance for the noise.

A more realistic implementation involves estimating the measurement variance, once in practice the variance cannot be determined before the operation and it must be estimated on-the-fly. From the derivations in this section we conclude that the variance $\sigma^2_k$ is equal to the expected value of $r_k^2 - E[r_k^2]$, which means $E[(r_k^2 - E[r_k^2])] = \sigma^2_k$. Assuming that the process $r_k$ is WSS and its fourth order moment is finite, as well as that the measurements $r_k$ are i.i.d., then one can use the ergodicity to state that a sample mean of $r_k^2 - E[r_k^2]$ converges with probability 1 to the variance of the process $z_k$, which is $\sigma^2_k$.

To build this sample mean one has to use a sample mean for the value of $r_k$ as well, which is $\hat{r}_k = \frac{1}{n} \sum_{i=0}^{n} r_k(i)$. Now, for the sample mean of $r_k^2 - E[r_k^2]$, one just has to calculate

$$\hat{\sigma}^2_k = \frac{1}{n} \sum_{i=0}^{n} (r_k(i) - \hat{r}_k(i))^2 \tag{18}$$

by the ergodic theorem the term $\frac{1}{n} \sum_{i=0}^{n} r_k^2(i)$ converges to $||p_k - \theta||^2 + \sigma^2_k$ and the term $\frac{1}{n} \sum_{i=0}^{n} r_k(i)$ converges to $E[r_k^2]$, which is the same as $||p_k - \theta||^2$.

Since for $n$ reasonably large $\hat{\sigma}^2_k \approx \sigma^2_k$, the input signal to the AN is given by $w_k \approx ||p_k - \theta||^2 + \hat{z}_k$, where $\hat{z}_k$ is a WSS random process with zero mean. As time progresses the effects of the initial conditions of the AN disappear, and it is expected that the difference between the cases with known variance and estimated variance vanish.

The variance estimator can be implemented recursively as

$$\hat{r}_k(i) = \hat{r}_k(i - 1)(i - 1) + r_k(i) \tag{19}$$

$$\hat{\sigma}^2_k(i) = \frac{\hat{\sigma}^2_k(i - 1)(i - 1) + (r_k(i) - \hat{r}_k(i))^2}{i} \tag{20}$$

Simulations in this section follow (13) and present the known variance case and the variance estimator case for Gaussian white process with $E[z_k] = 0$ and $\sigma^2 = 0.01$.

Figure 2 shows the ensemble average of the global MSD curve when the variance is known a priori. It also shows one realization of the estimate $w_{k,i}$ for some node in the $N = 10$ scenario.

The next simulation takes into account the effects of the variance estimation process in the performance of the whole system. Figure 3 shows the ensemble average of the global MSD curve, and it also shows a realization for the estimate $w_{k,i}$ for some node in the $N = 10$ scenario. Note that the effects of the variance estimator do not affect the AN performance considerably.

Previous simulations are repeated with a noise variance of $\sigma^2_k = 1$. Figure 4 shows the known variance case and Figure 5 depicts the case with the variance estimator, suggesting that there is no loss in the AN performance with the use of the variance estimator $\hat{\sigma}_k$ in the loop.

V. TRACKING ABILITY

The natural tracking ability of ANs [1] allows for following a moving object. The signal model used was the norm-squared
proportional and the network was required to track a target that varies according to the dynamics

\[ \theta(i) = \Delta(\theta(i-1), i) \]

where \( \Delta : \mathbb{R}^D \times N \to \mathbb{R}^D \) is some function from both the last position of the target, which is the case when the trajectory is generated by some feedback, and the iteration.\(^6\) We assume that we can measure a signal \( r_k \) as in equation (1) but corrupted by noise. In addition, the position of each sensor is corrupted by a Gaussian white noise with zero mean and variance \( \sigma^2_k = 0.01 \). This is similar to including a noise term in the dynamical model.

Initially the dynamic model is a straight line

\[ \Delta(\theta(i-1), i) = \theta(i-1) + (0.01, 0.01) \]

with \( \theta(0) = (0, 3) \). Again the nodes were randomly deployed using a uniform distribution in the beginning of each trial over a \( 10 \times 10 \) grid, and then positions are perturbed by noise. Each node’s estimate was initialized at zero, and the additive noise in (6) was generated using a Gaussian distribution with \( \mathbb{E} z_k = 0 \) and \( \sigma^2_k = 0.01 \). Figure 6 shows how an AN with \( N = 10 \) nodes was able to follow the object over a constant trajectory almost perfectly.

As intuition suggests, tracking in the limit case with \( N = 3 \) nodes is poor, and performance for larger ANs is much better.

In the sequel, the dynamic model is modified to

\[ \Delta(\theta(i-1), i) = (3\cos(i \ast T), 2\sin(i \ast T)) \]

\(^6\)It can be seen as a time-varying discrete dynamic system.

which represents an ellipsis, where \( T = 0.01 \). With this example we want to show the AN capability of following a target that makes curves.

A fully distributed adaptive solution to the source localization problem from scalar measurements was developed for two signal models via an LMS-based AN. It is able to estimate the source position even with a few nodes, under high and low SNR.

The squared-norm case is implemented directly via signals \( d_k(i) \) and \( u_k,i \). The norm proportional case requires an auxiliary signal and the knowledge of the noise variance, which may be obtained via an online estimator with no apparent performance loss as compared to the known variance case.

Tracking of moving targets following a straight line and an ellipsis were presented. In the dynamic case the number of nodes for good tracking ability seems more critical.

VI. CONCLUSION

A fully distributed adaptive solution to the source localization problem from scalar measurements was developed for two signal models via an LMS-based AN. It is able to estimate the source position even with a few nodes, under high and low SNR.

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REFERENCES


