Adaptive MSER Reduced-Rank Decision Feedback Receiver for Large-Scale Multiple-Antenna Systems

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Abstract—In this paper, we propose a novel adaptive reduced-rank decision feedback (DF) receiver strategy based on the minimization symbol-error-rate (MSER) criterion for large-scale multiple-antenna systems. In this strategy, a reduced-rank method for general parameter estimation using an adaptive joint and iterative preprocessing, decimation and filtering (JPDF) scheme is proposed for DF receive processing. We design a pre-stored decimation matrix and develop conjugate gradient (CG) and modified conjugate gradient (MCG) algorithms for computing the parameters of the preprocessor and reduced-rank filter. A computational complexity analysis is carried out for the existing and proposed adaptive DF algorithms. Simulation results show that the proposed adaptive MSER-JPDF reduced-rank DF receive processing strategy can achieve a superior performance to conventional adaptive linear and DF receive processing algorithms.

Index Terms—Adaptive filtering, minimum-SER, reduced-rank techniques, DF receiver, large-scale multiple-antenna.

I. INTRODUCTION

The high demand for system performance and capacity in wireless networks has driven the development of numerous signal processing and communications techniques towards large-scale multiple-antenna systems, since a massive number of antennas can guarantee a dramatic increase in spectral and power efficiency [1]–[3]. The problem of detecting a desired user’s signal in a multiuser large-scale multiple-antenna system presents many signal processing challenges, including: the need for algorithms with the ability to process large-dimensional received data vectors, fast and accurate adjustment of system parameters, scalable computational complexity and the development of cost-effective interference mitigation schemes.

In this context, reduced-rank signal processing is a very powerful technique that has gained considerable attention in the last few years, since it can provide faster convergence speed, increased robustness to interference, and better tracking performance than full-rank techniques at an affordable complexity [4], [5]. A reduced-rank technique projects the large-dimensional received data vector onto a lower dimensional subspace. Numerous reduced-rank algorithms have been developed to design the projection matrix and the reduced-rank receive filter. Among these available methods, the designer may resort to techniques such as the early eigen-decomposition (EIG) approach [6], [7] and the promising multistage Wiener filter (MWF) scheme [8], [9]. Apart from these, a strategy based on the joint and iterative optimization (JIO) has been reported in [5], [10], [11], whereas algorithms using joint preprocessing, decimation and filtering (JPDF) schemes have been considered in [12], [13].

The state-of-the-art adaptive filtering design has traditionally been developed based on the minimum mean square error (MMSE) criterion, and it can readily be implemented using standard adaptive filter techniques [14]. However, the MMSE criterion is not the most appropriate metric from a performance viewpoint in digital communications since the measurement of transmission reliability is symbol-error-rate (SER) rather than mean square error (MSE). This has motivated the research for the alternative approach that aims to directly minimize the system’s SER [15]–[17]. The work in [16] appears to be the first approach to combine a reduced-rank algorithm with the minimization SER (MSER) criterion, however, the scheme is a hybrid between an EIG or an MWF approach, and an SER scheme in which only the reduced-rank filter is adjusted in an MSER fashion. The recently reported work in [17] investigated a novel adaptive reduced-rank strategy based on JIO of the receive filters according to the MSER cost function.

In this work, we propose an adaptive reduced-rank decision feedback (DF) receive processing strategy that incorporates the JPDF scheme optimized based on the MSER criterion for multiuser large-scale multiple-antenna systems. The proposed reduced-rank scheme employs a framework which contains a preprocessor, a decimation unit and a reduced-rank receive filter. We design a pre-stored decimation matrix and develop conjugate gradient (CG) and modified conjugate gradient (MCG) algorithms for computing the parameters of the preprocessor and reduced-rank filter. A computational complexity analysis is carried out for the existing and proposed adaptive DF algorithms. Simulation results show that the proposed adaptive MSER-JPDF reduced-rank DF receiver can achieve a superior performance to existing receiver techniques at a reduced complexity.

The rest of the paper is organized as follows. Section II presents the system model and the design problem statement.

1This work was supported by the National Science Foundation of China (NSFC) under Grant 61471319, Zhejiang Provincial Natural Science Foundation of China under Grant LY14F010013, and the 5th Generation Mobile Communication Program in China (863 Project) under Grant number 2014AA01A707.
Section III gives an overview of the JPDF reduced-rank scheme. The proposed adaptive MSER reduced-rank DF receive processing algorithms are described in Section IV along with a comparison of computational complexity. Simulation results are presented in Section V. Finally some conclusions are drawn in Section VI.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model

We consider a wireless communication scenario in which $K$ narrowband signals impinge on a uniform linear array (ULA) comprised of $L$ antennas. The sources are assumed to be in the far field with direction of arrivals (DOAs) $\theta_0, \ldots, \theta_{K-1}$. The $i$th snapshot’s received vector $\mathbf{r} \in \mathbb{C}^{L \times 1}$ can be modeled as

$$\mathbf{y}(i) = \mathbf{A}(\theta) \mathbf{b}(i) + \mathbf{n}(i), \quad (1)$$

where $\theta = [\theta_0, \ldots, \theta_{K-1}]^T \in \mathbb{R}^{K \times 1}$ is the vector with the DOAs of the signals, $\mathbf{A}(\theta) = [\mathbf{a}(\theta_0), \ldots, \mathbf{a}(\theta_{K-1})] \in \mathbb{C}^{L \times K}$ comprises the normalized signal steering vectors $\mathbf{a}(\theta) \in \mathbb{C}^{L \times 1}$

$$\mathbf{a}(\theta_k) = [1, e^{-2\pi j \frac{\lambda}{\lambda_c} \cos(\theta_k)}, \ldots, e^{-2\pi j (L-1) \frac{\lambda}{\lambda_c} \cos(\theta_k)}]^T, \quad (2)$$

where $k = 0, \ldots, K - 1$, $\lambda_c$ is the wavelength, $u$ ($u = \frac{\lambda}{\lambda_c}$ in general) is the inter-element distance of the ULA, and $(\cdot)^T$ stands for transpose. $\mathbf{b}(i) = [b_0(i), b_1(i), \ldots, b_{K-1}(i)]^T$ is the source data vector, where we assume that the symbols are independent and identically distributed (i.i.d) random variables with equal probability from the set $\{\pm 1 \pm j\}$. The vector $\mathbf{n}(i) \in \mathbb{C}^{L \times 1}$ is the complex-valued additive white Gaussian noise (AWGN) with $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma_n^2 \mathbf{I}$, where $\sigma_n^2$ denotes the variance and $\mathbf{I}$ is an identity matrix of order $L$. $E[\cdot]$ stands for expected value, and $(\cdot)^H$ denotes the Hermitian transpose.

Let us consider the structure of reduced-rank DF receivers. In this paper we adopt the parallel DF strategy, which first obtains the decision vector $\mathbf{b}_r(i)$ with a linear scheme and then utilizes the decisions of associated users to cancel the interference. In the following, without loss in generality, we only describe the procedure for detecting $\hat{b}_k(i)$ of user $k$. The detection of other users can be obtained accordingly. For user $k$, the $M \times 1$ input data vector $\mathbf{r}_k(i)$ is obtained by stacking the $L \times 1$ received vector $\mathbf{y}(i)$ and the $(K - 1) \times 1$ vector of decisions $\mathbf{b}_{T,k}(i)$ and can be described by

$$\mathbf{r}_k(i) = \begin{bmatrix} \mathbf{y}(i) \\ \mathbf{b}_{T,k}(i) \end{bmatrix}, \quad (3)$$

where $M = L + K - 1$ represents the number of samples for processing, $\mathbf{r}_k(i)$ takes the feedback into account and excludes the $k$th detected symbol to avoid canceling the desired symbol, $\mathbf{b}_{T,k}(i)$ is denoted by $\mathbf{b}_{T,k}(i) = [\hat{b}_0(i), \ldots, \hat{b}_{K-1}(i), \hat{b}_{K+1}(i), \ldots, \hat{b}_{K-1}(i)]^T$.

Then a reduced-rank interference suppression scheme is employed to process the input data vector $\mathbf{r}_k(i)$ in two stages. The first stage performs a dimensionality reduction via a decomposition of $\mathbf{r}_k(i)$ into a lower dimensional subspace. It exploits the low-rank nature of the data transmitted over user $k$ as follows

$$\mathbf{\bar{r}}_k(i) = \mathbf{S}_{D,k}^H(i)\mathbf{r}_k(i), \quad (4)$$

where $\mathbf{S}_{D,k}(i)$ denotes an $M \times D$ projection matrix which performs the dimensionality reduction. The second stage is carried out by a $D \times 1$ reduced-rank filter. The proposed reduced-rank DF output is

$$z_k(i) = \mathbf{\bar{w}}_k^H(i)\mathbf{S}_{D,k}^H(i)\mathbf{r}_k(i) = \mathbf{\bar{w}}_k^H(i)\mathbf{\bar{r}}_k(i), \quad (5)$$

where $\mathbf{\bar{w}}_k(i) = [\bar{w}_0(i), \bar{w}_1(i), \ldots, \bar{w}_{D-1}(i)]^T$ denotes the $D \times 1$ rank DF output. The detected symbols of the proposed reduced-rank DF receiver are obtained by

$$\hat{b}_k(i) = \mathbb{Q}\{z_k(i)\}, \quad (6)$$

where $\mathbb{Q}\{\cdot\}$ represents the quantization operation for the given constellation.

The basic problem in implementing the MSER reduced-rank receive processing scheme is how to effectively devise the projection matrix $\mathbf{S}_{D,k}(i)$ and the reduced-rank receive filter $\mathbf{\bar{w}}_k(i)$ [19]. In the following sections, we propose and investigate a new structure and adaptive algorithms for online estimation of these quantities. It should be noted that the projection matrix and the reduced-rank receive filter are updated jointly based on minimizing the SER cost function.

III. THE JPDF REDUCED-RANK SCHEME

In this section, we detail the JPDF reduced-rank DF receive processing scheme which contains a preprocessor, a decimation unit and a reduced-rank receive filter. We design the subspace projection matrix $\mathbf{S}_{D,k}(i)$ by considering preprocessing and decimation. The motivation is to reduce the complexity and improve the convergence speed of adaptive algorithms in large-scale multiple-antenna systems.

![Fig. 1: Proposed reduced-rank DF receiver structure](image-url)

Fig. 1 shows the structure of the JPDF reduced-rank DF receiver where an adaptive preprocessor and an adaptive reduced-rank filter are employed. The $M \times 1$ input vector $\mathbf{r}_k(i)$ is filtered by the preprocessing filter $\mathbf{p}_k(i) = [p_0(i), \ldots, p_{I-1}(i)]^T$ with $I$ being the length of the preprocessor and yields the preprocessed vector $\mathbf{\bar{r}}_k(i)$ with $M$ samples, which is expressed by

$$\mathbf{\bar{r}}_k(i) = \mathbf{P}_k^H(i)\mathbf{r}_k(i), \quad (7)$$
where the $M \times M$ Toeplitz convolution matrix $P_k(i)$ has shifted copies of $p_k(i)$ as described by
\[
P_k(i) = \begin{pmatrix}
p_0(i) & 0 & \ldots & 0 \\
\vdots & p_0(i) & \ldots & 0 \\
p_{i-1} & \vdots & \ddots & \vdots \\
0 & p_{i-1} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & p_0(i)
\end{pmatrix}.
\]

In order to facilitate the description of the scheme, let us express the vector $r_k(i)$ in a way that is suitable for algebraic manipulation as a function of the preprocessor $p_k(i)$:
\[
\tilde{r}_k(i) = P_k^H(i)r_k(i) = R_k(i)p_k(i),
\]
where the $M \times I$ matrix $R_k(i)$ with the samples of $r_k(i) = [r_0(i), \ldots, r_{M-1}(i)]^T$ has a Hankel structure [18] given by
\[
R_k(i) = \begin{pmatrix}
r_0(i) & r_1(i) & \ldots & r_{i-1}(i) \\
\vdots & \vdots & \ddots & \vdots \\
r_{M-1}(i) & r_{M-1+1}(i) & \ldots & r_{M-1}(i) \\
r_{M-1+1}(i) & \vdots & \ddots & \vdots \\
r_{M-2}(i) & r_{M-1}(i) & 0 & 0 \\
r_{M-1}(i) & 0 & 0 & 0
\end{pmatrix}.
\]

The dimensionality reduction is performed by a decimation unit with $D \times M$ decimation matrix $T$ that projects $\tilde{r}_k(i)$ onto $D \times 1$ vector $F_k(i)$, where $D$ is the rank. The $D \times 1$ vector $\bar{r}_k(i)$ is given by
\[
\bar{r}_k(i) = S_{D,k}(i)r_k(i) = TR_k(i)p_k(i),
\]
where $S_{D,k}(i)$ denotes the equivalent subspace projection matrix.

The elements of the decimation matrix only take the value 0 or 1. This corresponds to the decimation unit simply keeping or discarding the samples. We introduce the structure of the decimation matrix as follows,
\[
T = \begin{bmatrix}
t_1 & t_2 & \ldots & t_D \end{bmatrix}^T,
\]
where the $M \times 1$ vector $t_d$ denotes the $d$-th basis vector of the decimation unit, $d = 1, \ldots, D$, its structure is described by
\[
t_d = \begin{bmatrix}
0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0
\end{bmatrix}^T
\]
where $q_d$ is the number of zeros before nonzero element. Note that it is composed of a single 1 and $M-1$ 0s. We set the value of $q_d$ in a deterministic way which can be expressed as $q_d = \lfloor \frac{M}{D} \rfloor \times (d-1)$, where $\lfloor . \rfloor$ represents the floor function which returns the largest integer that is smaller than or equal to its argument. The simulation results will show that the proposed reduced-rank scheme with this decimation unit design method works very well.

IV. PROPOSED MSER-CG REDUCED-RANK ALGORITHM

In this section, we first formulate the theoretical MSER cost function for the reduced-rank DF receiver design. Then, we develop the MSER based adaptive CG and MCG algorithms for updating the preprocessor and the reduced-rank filters. Finally, we present a computational complexity analysis for the proposed and conventional adaptive DF algorithms.

A. MSER Cost Function

Let us consider the case with quadrature phase shift keying (QPSK) symbols. It should be noted that the following derivation is general and could operate with various modulation techniques. For the proposed JPDF reduced-rank scheme, the symbol error probability regarding user $k$ can be represented as
\[
\begin{align}
\mathcal{P}_e(\hat{w}_k(i), p_k(i)) &= \mathcal{P}_e^R(\hat{w}_k(i), p_k(i)) + \mathcal{P}_e^I(\hat{w}_k(i), p_k(i)) \\
&= \mathcal{P}_e^R(\hat{w}_k(i), p_k(i)) + \mathcal{P}_e^I(\hat{w}_k(i), p_k(i))
\end{align}
\]
where $\mathcal{P}_e(\hat{w}_k(i), p_k(i)) = \text{Prob}\{b_k(i) \neq \hat{b}_k(i)\}$ denotes the total SER, $\mathcal{P}_e^R(\hat{w}_k(i), p_k(i)) = \text{Prob}\{\Re[b_k(i)] \neq \Re[\hat{b}_k(i)]\}$ and $\mathcal{P}_e^I(\hat{w}_k(i), p_k(i)) = \text{Prob}\{\Im[b_k(i)] \neq \Im[\hat{b}_k(i)]\}$ denote the real part and imaginary part SERs, respectively. Here, $\Re[.]$ and $\Im[.]$ select the real and imaginary parts, respectively. The optimization problem is formulated to minimize an upper bound of the SER as follows [19]
\[
\min_{\hat{w}_k(i), p_k(i)} \mathcal{P}_e(\hat{w}_k(i), p_k(i))
\]
where $\mathcal{P}_e(\hat{w}_k(i), p_k(i)) = \mathcal{P}_e^R(\hat{w}_k(i), p_k(i)) + \mathcal{P}_e^I(\hat{w}_k(i), p_k(i))$. For small values of SER, the true SER $\mathcal{P}_e(\hat{w}_k(i), p_k(i))$ is quite close to the upper bound $\mathcal{P}_e(\hat{w}_k(i), p_k(i))$, that is to say, the solution obtained by minimizing (12) is equivalent to that of minimizing $\mathcal{P}_e(\hat{w}_k(i), p_k(i))$.

The decision variable of the receiver is drawn from a Gaussian mixture and can be redescibed as follows
\[
z_k(i) = w_k^H(i)A(\theta)\mathbf{b}(i) + w_k^H(i)n(i),
\]
where $w_k(i) = P_k(i)T^T\tilde{w}_k(i)$ and the first term $\hat{z}_k(i) = w_k^H(i)A(\theta)\mathbf{b}(i)$ is the noise free receiver output. A single-sample estimate of the true probability density function (PDF) for $z_k(i)$ is given by
\[
p(z_k(i)) = \frac{1}{\sqrt{2\pi\rho}} \exp\left(-\frac{(z_k(i) - \hat{z}_k(i))^2}{2\rho^2}\right)
\]
where $\rho$ is the radius parameter of the kernel density estimate [20]. We can write the error probability for the real part of user $k$ as
\[
\mathcal{P}_e^R(\hat{w}_k(i), p_k(i)) = Q\left(\frac{\text{sgn}\{\Re[b_k(i)]\}\Re[\hat{z}_k(i)]}{\rho(w_k^H(i)w_k(i))^2}\right),
\]
where $\text{sgn}\{\cdot\}$ is the signum function and $Q(.)$ is the Gaussian error function. Similarly, $\mathcal{P}_e^I(\hat{w}_k(i), p_k(i))$ can be evaluated.
using the imaginary part
\[ P_c^I (\hat{w}_k(i), p_k(i)) = Q \left( \frac{\text{sgn}\{3[b_k(i)]\} 3[\hat{z}_k(i)]]}{\rho (w^H_k(i) w_k(i))^\frac{3}{2}} \right) \]  

(16)

B. Proposed Adaptive Algorithms

1) Conjugate Gradient (CG) Algorithm: Firstly, we should derive the gradient terms for the preprocessing and reduced-rank filters, which can be computed as
\[ \frac{\partial P_c^R}{\partial p_k(i)} = \frac{1}{2\pi \rho} \text{exp} \left( -\frac{\| \hat{z}_k(i) \|^2}{2\rho^2 w^H_k(i) w_k(i)} \right) \times \text{sgn}(\| b_k(i) \|) \left( -B^H(i) R[\hat{r}_k(i)] \right) \left( w^H_k(i) w_k(i) \right)^{\frac{3}{2}} \]

(17)
\[ \frac{\partial P_c^I}{\partial p_k(i)} = \frac{1}{2\pi \rho} \text{exp} \left( -\frac{3\| \hat{z}_k(i) \|^2}{2\rho^2 w^H_k(i) w_k(i)} \right) \times \text{sgn}(3[b_k(i)]) \left( -B^H(i) 3[\hat{r}_k(i)] \right) \left( w^H_k(i) w_k(i) \right)^{\frac{3}{2}} \]  

(18)
\[ \frac{\partial P_c^R}{\partial w_k(i)} = \frac{1}{2\pi \rho} \text{exp} \left( -\frac{\| \hat{z}_k(i) \|^2}{2\rho^2 w^H_k(i) w_k(i)} \right) \text{sgn}(\| b_k(i) \|) \left( -T^H_k(i) R[\hat{r}_k(i)] \right) \left( w^H_k(i) w_k(i) \right)^{\frac{3}{2}} \]  

(19)
\[ \frac{\partial P_c^I}{\partial w_k(i)} = \frac{1}{2\pi \rho} \text{exp} \left( -\frac{3\| \hat{z}_k(i) \|^2}{2\rho^2 w^H_k(i) w_k(i)} \right) \text{sgn}(3[b_k(i)]) \left( -T^H_k(i) 3[\hat{r}_k(i)] \right) \left( w^H_k(i) w_k(i) \right)^{\frac{3}{2}} \]  

(20)

where the \( M \times I \) matrix \( B(i) \) is a Hankel matrix with the elements of \( T^T \hat{w}_k(i) \). In practice, we develop a CG-based adaptive algorithm that adjusts the parameters based on the minimization of the SER cost function. Since a stochastic gradient (SG) algorithm may converge slowly, and a recursive least square (RLS) algorithm is computationally expensive. The CG method offers a better alternative. The preprocessing and reduced-rank receive filters are jointly optimized according to the MSER criterion. The algorithm has been devised to start its operation in the training (TR) mode, and then to switch to the direction-directed (DD) mode. The proposed MSER-JPDF reduced-rank CG algorithm is summarized in Table I. Note that we drop the index \( k \) for notation simplicity.

The step size \( \mu_p, \mu_w \), the number of iterations \( N \) and the kernel width \( \rho \) are algorithmic parameters that should be set appropriately in order to attain an adequate performance in terms of both the convergence rate and steady-state SER performance.

<table>
<thead>
<tr>
<th>TABLE I: The Proposed Adaptive CG Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>For each user:</td>
</tr>
<tr>
<td>Initialization:</td>
</tr>
<tr>
<td>( p^{(0)} = [1, 0, \ldots, 0]^T, \quad \psi^{(0)} = [1, 0, \ldots, 0]^T. )</td>
</tr>
<tr>
<td>( \mu_p ) and ( \mu_w ) are small positive values.</td>
</tr>
<tr>
<td>For each time instant ( i = 0, 1, 2, \ldots )</td>
</tr>
<tr>
<td>Update the preprocessor:</td>
</tr>
<tr>
<td>Set ( n = 0, g_p^{(n)}(i) = 0, d_p^{(n)}(i) = \frac{\partial P_c^R(\hat{w}, p)}{\partial p}; \quad g_w^{(n)}(i) = \frac{\partial P_c^I(\hat{w}, w)}{\partial w} ),</td>
</tr>
<tr>
<td>For ( n = 0, \ldots, N - 1 )</td>
</tr>
<tr>
<td>( p^{(n+1)}(i) = p^{(n)}(i) - \mu_p g_p^{(n)}(i), )</td>
</tr>
<tr>
<td>( d_p^{(n+1)}(i) = \frac{\partial P_c^R(\hat{w}^{(n+1)}(i), \hat{w}^{(n+1)}(i))}{\partial p}; \quad g_w^{(n+1)}(i) = \frac{\partial P_c^I(\hat{w}^{(n+1)}(i), \hat{w}^{(n+1)}(i))}{\partial w} )</td>
</tr>
<tr>
<td>( \phi^{(n)}(i) =</td>
</tr>
<tr>
<td>( \phi^{(n)}(i) = \phi^{(n)}(i) d_p^{(n)}(i) - d_p^{(n+1)}(i), \quad n = n + 1. )</td>
</tr>
<tr>
<td>After ( N ) iterations:</td>
</tr>
<tr>
<td>Update the reduced-rank filter:</td>
</tr>
<tr>
<td>Set ( n = 0, g_p^{(0)}(i) = 0, d_p^{(0)}(i) = \frac{\partial P_c^R(\hat{w}, p)}{\partial p}; \quad g_w^{(0)}(i) = \frac{\partial P_c^I(\hat{w}, w)}{\partial w} ),</td>
</tr>
<tr>
<td>For ( n = 0, \ldots, N - 1 )</td>
</tr>
<tr>
<td>( \hat{w}^{(n+1)}(i) = \hat{w}^{(n)}(i) - \mu_p g_w^{(n)}(i) ),</td>
</tr>
<tr>
<td>( d_w^{(n+1)}(i) = \frac{\partial P_c^R(\hat{w}^{(n+1)}(i), \hat{w}^{(n+1)}(i))}{\partial p}; \quad g_w^{(n+1)}(i) = \frac{\partial P_c^I(\hat{w}^{(n+1)}(i), \hat{w}^{(n+1)}(i))}{\partial w} )</td>
</tr>
<tr>
<td>( \phi^{(n)}(i) =</td>
</tr>
<tr>
<td>( \phi^{(n)}(i) = \phi^{(n)}(i) d_p^{(n)}(i) - d_w^{(n+1)}(i), \quad n = n + 1. )</td>
</tr>
<tr>
<td>After ( N ) iterations:</td>
</tr>
<tr>
<td>( \hat{w}^{(n+1)}(i) = \hat{w}^{(n+1)}(i) )</td>
</tr>
</tbody>
</table>

2) Modified Conjugate Gradient (MCG) Algorithm: As stated in [15], the SER surface is highly nonlinear, occasionally. The search direction \( d_p \) and \( d_w \) may no longer be a good approximation to the CG direction when the iteration index becomes large. It is thus important to periodically reset the direction vector to the true gradient in order to ensure the convergence of the algorithm for sample-by-sample update. Motivated by this a non-reset method can be employed for the computation of \( \phi_w \) which is given by
\[ \phi_w = \frac{\left( d_o(i) - d_o(i-1) \right)^H d_o(i)}{||d_o(i-1)||^2} \quad o \in \{ p, w \}. \]

It can be used to improve performance. The simulation results have shown that this modified CG algorithm performs better because \( g_p^{(i)}(i) = \hat{g}_p^{(i)}(i) \) and \( g_p^{(i)}(i) = \hat{g}_w^{(i)}(i) \) will not be exactly zero. Table II below shows an implementation of this algorithm.

C. Computational Complexity

In Table III, we show the number of additions and multiplications of the proposed adaptive MSER-JPDF reduced-rank DF receive processing algorithms, the existing adaptive MWF reduced-rank DF receive processing algorithms and the full-rank adaptive DF receive processing algorithms. In the case of large-scale antenna systems, the parameters \( D \) and \( I \) are chosen much smaller than \( M \), which will result in a substantial complexity saving. In particular, for a configuration with \( L = 60, K = 8, N = 3, D = 12 \) and \( I = 10 \), the numbers of multiplications and additions for the proposed MSER-JPDF-MCG algorithm are given by 5510 and 6750, respectively.
TABLE II: The Proposed Adaptive MCG Algorithm

For each user:
Initialization:
\[
p(0) = [1, 0, \ldots, 0]^T, \quad w(0) = [1, 0, \ldots, 0]^T.
\]
\[
\mu_w \text{ and } \mu_p \text{ are small positive values.}
\]
Set \( g_p(0) = 0, g_w(0) = 0, d_p(0) = \frac{\partial P_p(w,p)}{\partial w} \quad |w=\hat{w}(0), p=p(0)|. \)

and \( d_w(0) = \frac{\partial P_p(w,p)}{\partial p} \quad |p=\hat{p}(0), w=w(0)|. \)

For each time instant \( i = 0, 1, 2, \ldots \)
Update the preprocessor:
\[
p(i + 1) = p(i) - \mu_p g_p(i),
\]
\[
d_p(i + 1) = \frac{\partial P_p(w,p)}{\partial w} \quad |w=\hat{w}(i), p=p(i+1)|.
\]
\[
\phi_p(i + 1) = \phi_p(i + 1) g_p(i + 1)
\]
\[
\phi_p(i + 1) = \phi_p(i + 1) g_p(i + 1) - d_p(i + 1).
\]
Update the reduced-rank filter:
\[
w(i + 1) = \hat{w}(i) - \mu_w g_w(i),
\]
\[
d_w(i + 1) = \frac{\partial P_p(w,p)}{\partial p} \quad |p=\hat{p}(i+1), w=w(i+1)|.
\]
\[
\phi_w(i + 1) = \phi_w(i + 1) g_w(i + 1) - d_w(i + 1).
\]

TABLE III: Computational Complexity of Algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of operations per symbol</th>
<th>Multiplications</th>
<th>Additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-Rank MSER-SG</td>
<td>(3M + 8)</td>
<td>4M</td>
<td></td>
</tr>
<tr>
<td>Full-Rank MSER-CG</td>
<td>(4MN + 8N + M)</td>
<td>5MN + M</td>
<td></td>
</tr>
<tr>
<td>MSER-MWF SG</td>
<td>(6MD + M^2)</td>
<td>6MD</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M + D + 10</td>
<td>(M^2 - 2D)</td>
<td></td>
</tr>
<tr>
<td>MSER-MWF CG</td>
<td>(6MDN)</td>
<td>6MDN</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+DN + 8N</td>
<td>-2DN - MN</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+M^2 + M + 2</td>
<td>+M^2 + 2M</td>
<td></td>
</tr>
<tr>
<td>MSER-JPDF SG</td>
<td>(4D^2 + 6MD)</td>
<td>(2D^2 + 8MD)</td>
<td>2D + 2I</td>
</tr>
<tr>
<td></td>
<td>+3D + I + 20</td>
<td>-2D + 2I</td>
<td></td>
</tr>
<tr>
<td>MSER-JPDF CG</td>
<td>(4D^2N)</td>
<td>(2D^2N)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+6MDN + 3DN</td>
<td>+8MDN - 2DN</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+IN + 20N + D + I</td>
<td>+2IN + 2D + 2I</td>
<td></td>
</tr>
<tr>
<td>MSER-JPDF MCG</td>
<td>(4D^2 + 6MD)</td>
<td>(2D^2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+5D + 3I + 20</td>
<td>+8MD + 3I</td>
<td></td>
</tr>
</tbody>
</table>

The numbers of multiplications and additions for the proposed MSER-JPDF-CG algorithm are given by 16420 and 20192, respectively. And for the MSER-MWF-CG algorithm, they are 19090 and 18822. Compared to the existing reduced-rank DF receive processing algorithms, the proposed MSER-JPDF-MCG reduced-rank DF receive processing algorithms reduce the computational complexity significantly.

V. SIMULATIONS

In this section, we evaluate the SER performance of the proposed MSER-JPDF reduced-rank algorithm and compare it with existing algorithms, i.e., the full-rank and the reduced-rank MWF techniques. These algorithms are conducted with the proposed DF receivers and linear receivers. SG and CG adaptive methods are investigated respectively. On the receiver side, the uniform linear array is composed of \(L = 60\) sensor elements. In each simulation, the DOAs of the users are randomly generated with a uniform distribution between 0 and 180 degrees. The algorithms process 200 symbols in the training mode.

A. Compared with Linear Receivers

Firstly, we compare the proposed adaptive DF receive processing algorithms with the linear receive processing algorithms. Fig. 2 shows the SER performance versus the number of received symbols for the proposed MSER-JPDF reduced-rank DF receive processing algorithms, the MSER-JPDF reduced-rank linear receive processing algorithms, and the conventional linear adaptive receive processing algorithms.

We set the number of users \(K = 8\), \(SNR = 10dB\), \(D = 12\) and \(I = 10\). This figure illustrates that the proposed reduced-rank DF receive processing algorithms, including both the CG and MCG methods, have better steady-state performance and faster converge performance than the adaptive linear receive processing algorithms.

B. Under Adaptive DF Receivers

Secondly, we concentrate on the adaptive DF receivers. Fig. 3 evaluates the SER performance versus the number of received symbols for the proposed MSER-JPDF reduced-rank DF receive processing algorithms and the conventional full-rank and reduced-rank DF receive processing algorithms. We set the number of users \(K = 8\), \(SNR = 10dB\), \(D = 12\) and \(I = 10\). The results illustrate that the proposed reduced-rank DF receive processing algorithms have better convergence performance than the conventional full-rank and reduced-rank DF receive processing algorithms.

In Fig. 4, we investigate the steady-state SER performance versus the number of users \(K\) and SNR for the analyzed adaptive DF receive processing algorithms. We can find that the best performance is achieved by the proposed MSER-JPDF reduced-rank DF receiver with the MCG algorithm, followed by the proposed reduced-rank DF receiver with the CG and SG algorithms and the conventional adaptive DF receive processing algorithms. In particular, the proposed MSER-JPDF MCG reduced-rank DF algorithm can lead to a
3dB gain in SNR and support 4 more users in comparison with the adaptive MSER-MWF CG reduced-rank DF algorithm at the SER level of $3 \times 10^{-3}$.

VI. CONCLUSIONS

In this paper, we proposed an adaptive MSER-JPDF reduced-rank DF receive processing strategy for large-scale multiple-antenna systems. Furthermore, we developed CG-based methods to update the parameters of the preprocessor and reduced-rank filters. A computational complexity comparison has been carried out to show its advantage. Simulation results showed that the proposed algorithms significantly outperform the existing algorithms.

REFERENCES