Analytical Modeling of Downlink Power Control in Two-tier Femtocell Networks

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Abstract—Despite the advantages of deploying femtocells into cellular networks, the consequent cross-tier interference has seriously limited the network performance. In the downlink, specially, the strong interference from Femtocell Base Stations (FBSs) to nearby macrocell users (MUEs) causes significant coverage holes. Applying different power control techniques has been heavily discussed in 3GPP as an efficient approach for handling dominant interference scenarios. Instead of relying on system level simulations, this paper proposes an analytical model for evaluating downlink power control at femto-tier based on spatial Poisson Point Process (PPP). Tractable closed-form expressions for the coverage probability of both macrocells and femtocells are obtained. We also derive the average achievable rates of MUEs and femto users (FUEs). Some insights into the power control design are given and can be helpful in practical network deployment.

Index Terms—Femtocell Network, Downlink, Power Control, Poisson Point Process, Coverage Probability, Average Rate.

I. INTRODUCTION

As the demand for mobile data service increase significantly, cellular networks are now facing a paradigm transform from voice-centric to data-centric. It is apparent that traditional cellular network deployments are unable to support such a data explosion through past methods, like increasing the amount of spectrum or the amount of base stations [1]. In order to support this galloping demand for data traffic, a heterogeneous network structure is proposed by Third Generation Partnership Project (3GPP) [2], which consists of macrocells, picocells, relay stations and as well as femtocells. In this paper we consider a two-tier network with femtocells underlaying. Femtocells transmit at a very low power and provide better service for indoor consumers. However, despite the advantages femtocells have, they also cause more complicate interference management problems, especially arising from the cross-tier interference.

In this paper the femtocells adopt closed access policy, in which only licensed home users within femtocell range can be served by their own FBSs. With closed access, cross-tier interference from femtocells may significantly damage the SINR at the MUEs. For example, the strong interference from FBSs to nearby MUEs cause significant coverage holes. The motivation behind this paper is ensuring that the performance experienced by MUEs remain unaffected by the deployment of femtocells.

Among different interference management techniques proposed in [3], [4], power control has been considered as an effective approach for mitigating interference. However, in most previous work about downlink power control, the performance is evaluated through system level simulations [5]- [8], which has not led to general or tractable results. In this paper, we model the base stations in both tiers as two PPPs with different densities. This modeling approach has recently been widely used to analyze different problems in communication systems due to the ability to obtain tractable results [9]- [13]. Using this model, a downlink power control technique is proposed at the femto-tier to mitigate the interference to nearby MUEs, while at the same time guarantee the QoS of FUEs.

The main contributions of our work are summarized as follows:

1. General downlink power control model. In this paper we propose a tractable downlink power control model, in which the distribution of MBSs and FBSs are modeled as two PPPs with different densities. The femtocell adopts closed access policy, and a power control technique is applied in femto-tier to mitigate the interference to nearby MUEs. The power an FBS uses in the downlink channel depends on the received power of the closest MBS. This is a very general downlink power control model and can be extended to LTE-Advanced standards.

2. Closed-form coverage probability, average achievable rate. Assume that MUEs are associated with the closest MBS, and FUEs are uniformly distributed over femtocell ranges, we derive simple expressions for coverage probability of both MUEs and FUEs, which involve quickly computable integrals. In some special cases (e.g. interference limited), they can be further simplified to closed-form expressions. Closed-form lower bound of the average achievable rate for MUEs and FUEs are also obtained. These tractable results offer us a better understanding of the interactions between various system parameters.

3. Insights into the design of power control technique. We obtain some insights into the design of power control technique to guarantee the QoS of both MUEs and FUEs. Some important power control parameters are considered and closed-form expressions are derived.

The rest part of this paper is organized as follows. Sec. II introduces the system model. Sec. III and Sec. IV analyze the coverage probability and average achievable rate of both
MUEs and FUEs. Sec. V discusses the choice of power control parameters and Sec. VII concludes this paper.

II. SYSTEM MODEL

A. Two-tier Femtocell Network

A two-tier femtocell network consists of macrocells and femtocells as shown in Fig. 1. The locations of MBSs and FBSs are modeled as independent spatial PPP \( \Phi_m \) of density \( \lambda_m \) and \( \Phi_f \) of density \( \lambda_f \) respectively. We assume each MUE is associated with the closest MBS. The femtocells adopt closed access policy with a fixed coverage range \( R_f \). FUEs constitute a uniform distribution with their femtocell coverage.

The standard path loss propagation model is adopted with path loss exponent \( \alpha > 2 \). The small-scale fading between a BS \( z \) and the typical user is assumed to be Rayleigh fading and denoted by \( g_z \), namely \( g_z \sim \text{Exp}(1) \). The noise power is assumed to be constant additive with value \( \sigma^2 \). The wall penetration loss factor experienced through a femtocell is denoted by \( \Phi \).

As MUEs near FBSs are the main victims of femtocell interference, their performance becomes the most important measurement of a power control technique. We assume the MUEs near an FBS access the same closest MBS with the FBSs. The desired signal power at these MUEs is approximately equal to \( r_f^{-\alpha} \). Hence, the SIR (consider only the one MBS and the one FBS) of a typical MUE at a distance \( r \) from the nearby FBS is

\[
\text{SIR}_{\text{MUE}}(r) = \frac{r_f^{-\alpha}}{P_f r^{-\alpha} w}.
\]

In order to avoid causing strong interference to nearby MUEs, we set a protection radius \( r_0 \), beyond which the SIR is greater than a threshold \( T_0 \), namely \( \text{SIR}_{\text{MUE}}(r_0) = T_0 \). Hence, we obtain the transmit power of FBSs as

\[
P_f(r_f) = \frac{r_f^{-\alpha}}{T_0 w r_0^{-\alpha}}.
\]

(1)

\( r_f \) is a random variable with probability density function (pdf) \[9\] as follows:

\[
f_r(r_f) = e^{-\pi \lambda_f r_f^2} 2\pi \lambda_f r_f.
\]

(2)

Moreover, the random variables \( \{P_f\}_{z \in \Phi_f} \) can be assumed to be independent according to \[14\].

B. Power Control at Femto-tier

Referring the different power control techniques introduced in \[4\], the main idea is to decide the transmit power of an FBS according to the strongest received power \( P_r^{\text{max}} \) from macro-tier. In this paper, we assume the strongest received power is a long term averaged result and so comes from the closest MBS at a distance \( r_f \) from the typical FBS, namely \( P_r^{\text{max}} = w r_f^{-\alpha} \).

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\]

(2)

Moreover, the random variables \( \{P_f\}_{z \in \Phi_f} \) can be assumed to be independent according to \[14\].

It is important to notice that \( P_f \to \infty \) when \( r_f \to 0 \), which is obviously impossible. Therefore, we define \( P_f(r_f) = P_f(1) \) when \( r_f < 1 \). The expectation of \( P_f \) is derived as follows:

\[
E[P_f] = \int_0^1 f_r(r_f) P_f(1) dr_f + \int_1^\infty f_r(r_f) P_f(r_f) dr_f
\]

\[
= \frac{1}{b} \left( (1 - e^{-\pi \lambda_m}) + (\pi \lambda_m)^{\alpha/2} \Gamma\left(1 - \frac{\alpha}{2}, \pi \lambda_m\right) \right)
\]

where \( b = T_0 w r_0^{-\alpha} \), \( \Gamma(n, x) = \int_x^\infty e^{-t} t^{n-1} dt \) is the Incomplete Gamma Function. Generally, \( \lambda_m \) is the order of \( 1e - 4 \), which makes \( 1 - e^{-\pi \lambda_m} \) close to 0. So we simplify the expression of \( E[P_f] \) as:

\[
E[P_f] = \frac{(\pi \lambda_m)^{\alpha/2}}{b} \Gamma\left(1 - \frac{\alpha}{2}, \pi \lambda_m\right).
\]

(3)

Similarly, in some places of this paper, we directly evaluate the integral from \( r_f = 1 \) to \( \infty \) when expectations are computed over \( P_f \) for simplification.

III. COVERAGE PROBABILITY

In this section we derive the coverage probabilities of MUEs and FUEs respectively. The probability of a typical user within coverage is defined as \( \text{CP}(T, \alpha) \equiv P(\text{SINR} > T) \).

A. Macrocell UEs

Without loss of generality we assume the typical MUE under consideration is located at the origin.

Theorem 1: The coverage probability of an MUE is given by

\[
\text{CP}_m(T, \alpha, \sigma^2) = \pi \lambda_m \cdot \int_0^\infty e^{-\pi \lambda_m v(1+\rho(T, \alpha)+C(\alpha)(T/T_0)^2/r_0^2\Gamma(0,\pi \lambda_m)\lambda_f)} - T \sigma^2 v^{-\alpha/2} dv
\]

where \( \rho(T, \alpha) = T^{2/\alpha} \int_{T^{-1/2}}^\infty \frac{1}{\Gamma(1+u^2/2)} du \) and \( C(\alpha) = 2\pi^2 \csc(2\pi/\alpha). \)

Proof: See Appendix.
The integral in (4) can be computed through numerical methods. Furthermore, it can be simplified to closed-form in some special cases.

**Special Case when \( \alpha = 4 \):** In this case, \( \rho(T, 4) = \sqrt{T}(\pi/2 - \arctan(1/\sqrt{T})) \), and \( C(4) = \pi^2/2 \). The coverage probability of an MUE can be derived according to the equation

\[
\int_0^\infty e^{-px} e^{-q x^2} \, dx = \sqrt{\frac{\pi}{q}} \exp\left(\frac{p^2}{4q}\right) Q\left(\frac{p}{\sqrt{2q}}\right)
\]

where \( Q(x) = \frac{1}{\sqrt{\pi}} \int_x^\infty \exp(-y^2/2) \, dy \) is the standard Gaussian tail probability. The simplified expression can be derived by denoting \( p = \pi \lambda_m (1 + \rho(T, 4) + \pi^2/2 - r_0^2) \sqrt{T}/T_0 \Gamma(0, \pi \lambda_m \lambda_f) \) and \( q = T \sigma^2 \), which is omitted here because of its cumbrance.

Specially, when the system is interference limited, namely \( \sigma^2 = 0 \), we obtain

\[
CP_m(T, 4, 0) = \frac{1}{1 + \sqrt{T}(\pi/2 - \arctan(1/\sqrt{T})) + \pi^2/2 T \Gamma(0, \pi \lambda_m \lambda_f)}
\]

which is a remarkably simple closed-form expression. The last two items in the denominator represent the influence of macro-tier and femto-tier respectively. The macro-tier item depends only on the SIR threshold \( T \), while the femto-tier item depends on \( T, \lambda_m, \lambda_f \) and parameters related to the power control technique. From a practical perspective, there are three ways to improve the SIR of MUEs: (i) decrease the density of FBSs, which means less interference from femto-tier. (ii) Strengthen the power control, which means set a higher protection threshold \( T_0 \) or smaller protection radius \( r_0 \). (iii) increase the density of MBS. Although the increase in signal power is counter-balanced by the increase in interference power [9] at macro-tier, benefit can be found at femto-tier. Increasing \( \lambda_m \) will cause greater transmit power of FBSs as shown in equation (3), which means stronger interference from femto-tier, yet it is not enough to counteract the increase in signal power.

**B. Femtocell UEs**

In this paper we assume FUEs are uniformly distributed within femtocell coverage.

**Theorem 2:** The coverage probability of an FUE is given by

\[
CP_f(T, \alpha, \sigma^2) = E_{P_f} \left[ \frac{2}{R_f^2} \int_{0}^{R_f} e^{-Tr^\alpha \sigma^2 / P_f} \, dr \right] \exp\left(-C(\alpha) \left(\frac{T \sigma^2}{P_f}\right)^{2/\alpha} \lambda_m r^2 \left(1 + A(\lambda_m, \alpha, \lambda_f \pi)\right) \right) r^\gamma \right]
\]

where \( A(\lambda_m, \alpha) = T_0^{-2/\alpha} r_0^2 \Gamma(0, \pi \lambda_m) \).

**Proof:** See Appendix.

The expectation over \( P_f \) can be computed by replacing \( P_f \) with (1) and evaluate the expectation over \( r_f \). As the final expression is cumbersome, we do not display it here and explore some special cases directly:

**Lemma 1:** **Special Case when \( \sigma^2 = 0 \):** When the system is interference limited, the coverage probability can be simplified as follows

\[
CP_f(T, \alpha, 0) = \frac{\pi \lambda_m}{T^{2/\alpha} B} \ln\left(1 + \frac{T^{2/\alpha} B}{\pi \lambda_m}\right)
\]

where \( B = \lambda_m C(\alpha) w^{1/\alpha} R_f^2 \left(T^{2/\alpha} r_0^{-2} + \pi \lambda_f \Gamma(0, \pi \lambda_m)\right)\).

**Proof:** See Appendix.

**IV. AVERAGE ACHIEVABLE RATE**

In this section we derive the average achievable rate of MUEs and FUEs over their own cell regions per subchannel. The achievable rate is given by \( \ln(1 + \text{SINR}) \) in units of nats/Hz. Hence, the mean rate is given as follows:

\[
\tau = \frac{1}{T} \mathbb{E}[\ln(1 + \text{SINR})] = \int_{t>0} P(\text{SINR} > e^t - 1) \, dt.
\]

Noticing that we have already derived the SINR distribution of both MUEs and FUEs, general results of the average rate can be obtained immediately. Moreover, closed-form tractable expressions for the lower bound of the average rate of MUEs and FUEs can be derived in some special cases.

**A. Macrocell UEs**

**Theorem 3:** The average achievable rate of an MUE in case of \( \alpha = 4, \sigma^2 = 0 \) is lower bounded by

\[
\tau^\text{lower}_m = \pi Q - 2\ln(Q) + \frac{\pi^2}{1 + Q^2} \Gamma(0, \pi \lambda_m \lambda_f).
\]

**Proof:** Plugging the SINR distribution of MUEs (6) into (9) we have

\[
\tau = \int_{t>0} P(\text{SINR}_m > e^t - 1) \, dt
\]

\[
> \int_{t>0} \frac{1}{1 + \frac{T}{\pi Q} e^t - 1 + \frac{T}{2 \pi Q} r_0^2 \Gamma(0, \pi \lambda_m \lambda_f)} \, dt
\]

\[
\tau^\text{lower}_m = \int_{x>0} \frac{2x}{1 + x Q 1 + x^2} \, dx = \frac{\pi Q - 2\ln(Q)}{1 + Q^2}
\]

\( \tau^\text{lower}_m \) is a monotonically decreasing function of \( Q \) (\( Q \geq \pi/2 \)). The average achievable rate increases when \( \lambda_f \) decreases, when \( \lambda_m \) increases or when the power control is strengthen, which coincide with the three ways to improve the SIR of an MUE discussed in Sec. III

**B. Femtocell UEs**

**Theorem 4:** The average achievable rate of an FUE in case of \( \sigma^2 = 0 \) is lower bounded by

\[
\tau^\text{lower}_f = \frac{\pi^2 \lambda_m}{B} \ln\left(\frac{B}{\pi \lambda_m}\right)
\]

**Proof:** This simple expression of \( \tau^\text{lower}_f \) has the same form with function \( f(x) = \ln x / x \), which is not a monotonic function. The maximum value of \( f(x) \) is \( 1/e \) at \( x = e \). Hence, \( \tau^\text{lower}_f \) will not be greater than 1.163 nats/Hz.
V. POWER CONTROL DESIGN

Applying power control at femto-tier improves the performance of MUEs while degrades the performance of FUEs. The selection of the power control parameters \((T_0, r_0)\) is important to guarantee QoS requirement for both MUEs and FUEs. In this section, we consider only special cases when \(\alpha = 4\) and \(\sigma^2 = 0\).

Outage probability (OP) is an important metric for the performance experienced by MUEs as voice traffic is their main business. Outage probability is defined as the probability that the SINR is smaller than a threshold \(\gamma_0\).

\[
\text{OP}_m(\gamma_0) = \mathbb{P}(\text{SINR}_m < \gamma_0) \tag{13}
\]

which is the complement of MUEs coverage probability at \(T = \gamma_0\). The QoS requirement of MUEs is to guarantee the outage probability is less than a threshold \(\delta\).

\[
1 - \text{CP}_m(\gamma_0, 4, 0) \leq \delta.
\]

Hence, according to the special case (6) of Theorem 1 we derive

\[
T_0 \geq \left(\frac{\pi^2 \sigma^2}{1 - \delta} - \sqrt{\frac{\pi}{\sqrt{2}} - \arctan \left(\frac{\pi}{\sqrt{2}}\right)}\right)^2 \tag{14}
\]

Noticing that about 70% data traffic happens indoor [1], we propose to guarantee that the average rate in femtocells is greater than a threshold \(\tau_0\). The lower bound of the average rate achievable at femtocells has been derived in Sec. IV. The QoS requirement is satisfied when the lower bound of average rate is greater than \(\tau_0\).

\[
\gamma_f^{lower} \geq \tau_0.
\]

According to the curve shape of \(\ln x/x\), we first derive the two solutions \(s_1^*\) and \(s_2^*\) for equation \(\ln x/x = \tau_0/\pi\). Then we have

\[
\frac{s_1^*(\tau_0)}{\pi \lambda_m} \leq \frac{B}{\pi \lambda_m} \leq \frac{s_2^*(\tau_0)}{\pi \lambda_m}
\]

and thus

\[
\left(\frac{2{s_1^*(\tau_0)}}{\pi w R_f^2} - \pi \lambda_f \Gamma(0, \pi \lambda_m) r_0^2\right)^2 \leq T_0
\]

\[
\leq \left(\frac{2{s_2^*(\tau_0)}}{\pi w R_f^2} - \pi \lambda_f \Gamma(0, \pi \lambda_m) r_0^2\right)^2 \tag{15}
\]

(14) and (15) are the two QoS requirements about \(T_0\) we have to take into consideration when designing the power control. For example, using the parameters defined in Table I, we obtain the QoS requirement \(1.7e3 \leq T_0 \leq 5.2e3\). The protection radius \(r_0\) is set according to how large the area we would like to protect from the FBS interference. Generally, we set \(r_0\) at the radius of the femtocells \(R_f\) or a little larger.

VI. NUMERICAL RESULTS

Some numerical results are presented in this section with parameters in Table I. Fig. 2 shows the coverage probability of MUEs and FUEs according to both analytical results and Monte Carlo simulation. We set \(T_0\) at 3e3 according to its range derived in Sec. V. The theoretical curve coincides with the simulation curve well for both MUEs and FUEs, which corroborates the accuracy of our theoretical analysis.

Fig. 3 displays the average achievable rate of MUEs and FUEs as a function of femtocell density \(\lambda_f\) according to the lower bounds derived in Sec. IV. The lower bound of the average rate of MUEs is derived by enlarging the interference from macro-tier. Hence, the larger proportion the interference from macro-tier accounts of total interference, the greater the error is. We can see from the figure that the error between the theoretical lower bound and the numerical simulation of MUEs becomes smaller as \(\lambda_f\) increases. It is because interference from femto-tier accounts a larger and larger proportion. The curves of the theoretical lower bound and the numerical simulation of FUEs match each other well as shown in Fig. 3, which indicates the lower bound we derived is appropriate and very close to the actual expression.

Fig. 4 shows the influence of \(T_0\) on the outage probability of MUEs and the average rate of FUEs. The outage probability of MUEs is plotted with the blue curve while the average rate of FUEs is plotted with the red curve. We observe that both curves monotonically decrease with \(T_0\), which means increasing \(T_0\) causes smaller outage probability of MUEs while lower average rate of FUEs. When \(T_0 > 1.7e3\), the outage probability of MUEs is blow the threshold 0.1. When
$T_0 < 5.2e3$, the average rate of FUEs is above $\tau_0 = 0.8$ nats/Hz. The results match the range of $T_0$ we derived in Sec. V.

VII. CONCLUSIONS

In this paper we provide a general downlink power control model for a two-tier femtocell network. The transmit power of an FBS depends on the interference from the closest MBS. The power control technique at femto-tier intends to guarantee a small outage probability of MUEs and basic average achievable rate of FUEs at the same time. Tractable coverage probability and average rate are obtained for both MUEs and FUEs. Moreover, closed-form expressions are obtained in some special cases. Our analysis reveals the interaction among different parameters, and shed useful insights into the choice of network parameters and the design of power control. The accuracy of our theoretical analysis is corroborated by numerical results.

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APPENDIX

A. Proof of Theorem 1

Conditioning on the distance $r$ to the nearest MBS of the typical MUE, the coverage probability is

$$CP_m(T, \alpha) = \mathbb{E}_r[\Pr(\text{SINR}_m > T|r)]$$

$$= \int_{r>0} \mathbb{P}(\frac{g r^{-\alpha}}{\sigma^2 + I_m + I_f} > T|r)e^{-\pi \lambda_m r^2}2\pi \lambda_m r dr$$

where (a) follows from the PDF of $r$ shown in (2). Using the fact that $g \sim \text{Exp}(1)$, we derive

$$\mathbb{P}(\frac{g r^{-\alpha}}{\sigma^2 + I_m + I_f} > T|r) = e^{-T r^\alpha}\mathcal{L}_{I_m}(T r^\alpha)\mathcal{L}_{I_f}(T r^\alpha)$$

where $\mathcal{L}_{I_m}(s)$ and $\mathcal{L}_{I_f}(s)$ are the Laplace transforms of random variable $I_m$ and $I_f$ respectively. Referring [9] we obtain $\mathcal{L}_{I_m}(T r^\alpha)$ directly:

$$\mathcal{L}_{I_m}(T r^\alpha) = \exp(-\pi \lambda_m r^2 \rho(T, \alpha))$$

Defining $R_z$ as the distance from FBS $z$ to the typical MUE and $g_z$ as the small scale fading value, which follows an i.i.d. distribution of Exp(1), we derive the expression for $\mathcal{L}_{I_f}(s)$ below:

$$\mathcal{L}_{I_f}(s) \overset{(a)}{=} \mathbb{E}_{\Phi_f} \left( \prod_{z \in \Phi_f} \mathbb{E}_{P_f}[\frac{1}{1+s w P_f R_z^{-\alpha}}] \right)$$

$$\overset{(b)}{=} \exp \left( -2\pi \lambda_f \int_0^{\infty} (1 - \mathbb{E}_{P_f}[\frac{1}{1+s w P_f R_z^{-\alpha}}])vdv \right)$$

$$= \exp \left( -2\pi \lambda_f \mathbb{E}_{P_f} \left[ \int_0^{\infty} (1 - \frac{1}{1+s w P_f R_z^{-\alpha}})vdv \right] \right)$$

$$= \exp \left( -C(\alpha)\lambda_f (sw)^{2/\alpha} \mathbb{E}_{P_f}[P_f^{2/\alpha}] \right)$$

where (a) follows from the i.i.d. distribution of $g_z$ and $P_f^z$. (b) follows from the probability generating functional (PGFL) [15] of the PPP. Using (1) the expectation over $P_f$ is derived as follows:

$$\mathbb{E}_{P_f}[P_f^{2/\alpha}] = \mathbb{E}_{P_f}[(r_f^{-\alpha}/b)^{2/\alpha}] = b^{-2/\alpha} \mathbb{E}_{r_f}[r_f^{-2}]$$

$$= b^{-2/\alpha} \int_1^{\infty} r_f^{-2}e^{-\pi \lambda_m r_f^2}2\pi \lambda_m r_f dr_f$$

$$= \pi \lambda_m b^{-2/\alpha} \Gamma(0, \pi \lambda_m).$$

Plugging (19) into (18) and letting $s = T r^\alpha$, we derive

$$\mathcal{L}_{I_f}(T r^\alpha) = \exp \left( -C(\alpha)(T/T_0)^{2/\alpha} r_0^2 \Gamma(0, \pi \lambda_m \lambda_f \pi \lambda_m r^2) \right)$$

Using (16), (17) and (20) derives the result.

Fig. 3. Average Rate of MUEs and FUEs as a function of $\lambda_f$.

Fig. 4. Outage Probability of MUEs and Average Rate of FUEs as a function of $T_0$. 

B. Proof of Theorem 2

Conditioning on the distance \( r \) from a typical FUE to its FBS, the coverage probability is

\[
CP_f(T, \alpha) = \mathbb{E}_r[\mathbb{P}(\text{SINR}_f > T|r)]
\]

\[
= \frac{2}{R_f^2} \int_0^{R_f} \mathbb{P}(\frac{P_f g \rho^{-\alpha}}{\sigma^2 + I_m + I_f} > T|r) \, dr
\]  

(21)

where (a) follows from the uniform distribution of \( r \). Similar to (16) with the difference that \( P_f \) is a random variable, we derive

\[
\mathbb{P}(\frac{P_f g \rho^{-\alpha}}{\sigma^2 + I_m + I_f} > T|r)
\]

\[
= \mathbb{E}_{P_f}[e^{-T r^\alpha/\rho^2} \mathcal{L}_{I_m}(T r^\alpha/P_f) \mathcal{L}_{I_f}(T r^\alpha/P_f)]
\]  

(22)

As \( P_f \) is independent of \( I_f \), we can derive \( \mathcal{L}_{I_f}(T r^\alpha/P_f) \) directly by plugging \( s = T r^\alpha/P_f \) into (18). Noticing that the wall penetration loss becomes \( w^2 \), we have,

\[
\mathcal{L}_{I_m}(T r^\alpha/P_f) = \exp \left( -C(\alpha) \frac{(uT / P_f T_0)^{2/\alpha} r^2 \Gamma(0, \pi \lambda_m) \lambda f \pi \lambda_m r^2}{\sigma^2} \right)
\]  

(23)

The value of \( P_f \) depends on the distance of the closest MBS, which makes \( I_m \) related to \( P_f \). From the equation (3) we observe the closest MBS is located at \( r_f = r(t_0 P_f r_0^{-1/\alpha}) \). Hence, we derive [9]

\[
\mathcal{L}_{I_m}(T r^\alpha/P_f) = \exp \left( -2\pi \lambda_m \int_{r_f}^{\infty} \frac{1}{1 + (v/r)^{\alpha} P_f (T w)^{-1}} \, dv \right)
\]  

(24)

Here we make an approximation for \( \mathcal{L}_{I_m}(s) \) by setting the lower limit to 0, which results in a lower bound. Then the expression can be simplified as

\[
\mathcal{L}_{I_m}(T r^\alpha/P_f) = \exp \left( -2\pi \lambda_m \int_{0}^{\infty} \frac{1}{1 + (v/r)^{\alpha} P_f (T w)^{-1}} \, dv \right)
\]

\[
= \exp \left( -C(\alpha) (T w / P_f) p^{2/\alpha} \lambda_m r^2 \right).
\]  

(25)

Combining (21), (22), (23) and (25) gives the final result.

C. Proof of Lemma 1

Defining \( D = C(\alpha)(T r)^{2/\alpha} \lambda_m (1 + A(\lambda_m, \lambda) \alpha f \pi) \) for convenience, equation (7) can be simplified under interference limited assumption as:

\[
CP_f(T, \alpha) = \mathbb{E}_r \left[ e^{-D b^{2/\alpha} r^2 / \sigma^2} \cdot e^{-\pi \lambda_m r^2 / 2 \pi \lambda_m r f \sigma} \right]
\]

\[
= e^{\frac{\pi \lambda_m}{\pi \lambda_m + D b^{2/\alpha} r^2}} \ln \left( 1 + \frac{D b^{2/\alpha} R_f^2}{\pi \lambda_m} \right)
\]  

(26)

The proof is completed.

D. Proof of Theorem 4

The average achievable rate of an FUE is

\[
\tau_m = \int_{t > 0} \mathbb{P}(\text{SINR}_f > e^t - 1) \, dt
\]

\[
= \int_{t > 0} \frac{\pi \lambda_m}{e^{\sqrt{e^t - 1} B} - 1} \ln \left( 1 + \frac{B x}{\pi \lambda_m} \right) \frac{2}{1 + x^2} \, dx
\]

\[
\geq \frac{\pi \lambda_m}{B} \int_{0}^{\infty} \ln \left( 1 + \frac{B x}{\pi \lambda_m} \right) \frac{2}{1 + x^2} \, dx
\]

Referring the equation \( \int_0^{\infty} \ln(x) \, dx = \pi^2 \ln \pi \) in [16], we derive

\[
\tau_m > \frac{\pi^2 \lambda_m}{B} \ln \frac{B}{\pi \lambda_m}
\]  

(28)

The proof is completed.

REFERENCES


